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A Technique for Solving Multiscale Problems by Hybridizing Frequency and Time Domain Algorithms

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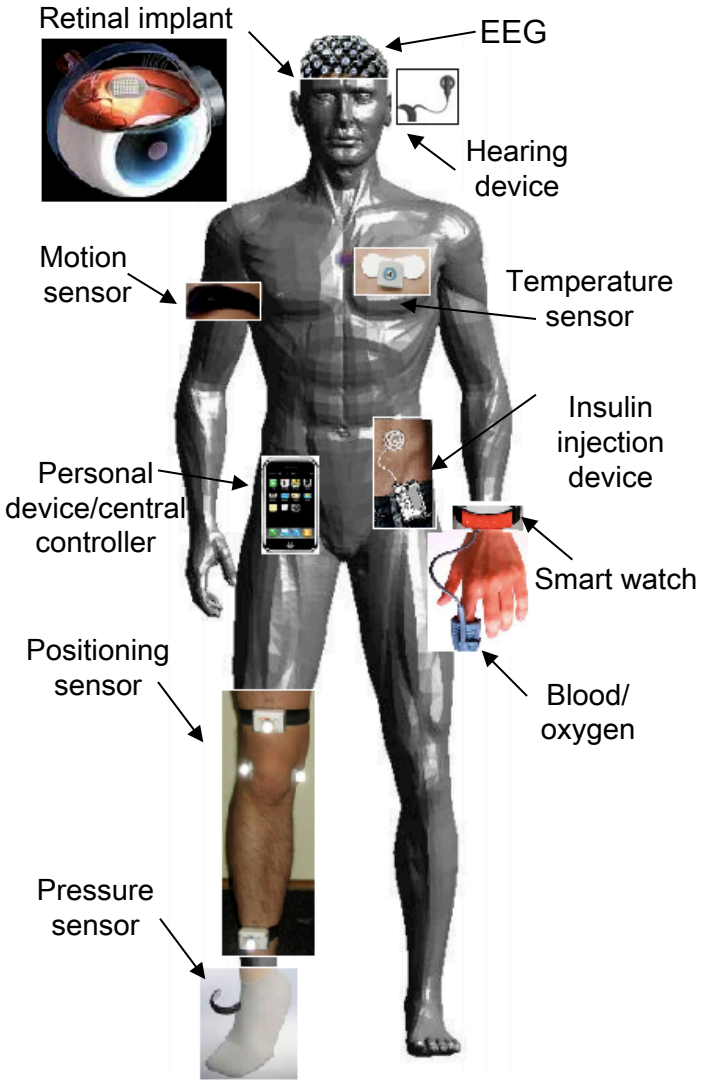
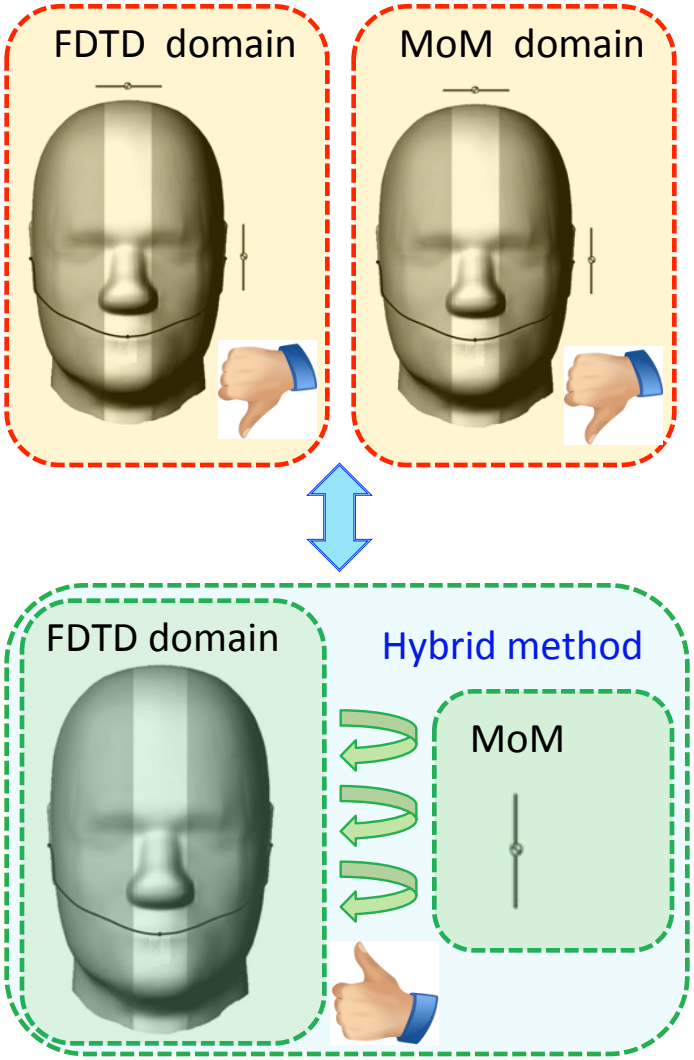
Introduction



- Motivation
- Hybridization of DM method with FDTD
  - ✓ Theory
  - ✓ Numerical results and considerations
- Proposed hybrid method
  - ✓ Theory
  - ✓ Numerical results and considerations
- Possible evolutions
- Conclusions

Motivation



- Large computational domain/inhomogeneous scatterers share the domain with objects with fine features (i.e. thin, curved/slanted wires)
- To model features whose dimensions do not coincide with the conventional Yee cell (both in terms of size, position and curvature)
- *Efficient*: do not increase the discretization around the object with fine features
- *General*: the wire does not necessarily need to lie on the FDTD grid
- *Simple*: does not require a significant change to the basic FDTD update equations
- *Accurate*: the local behavior of the fields is correctly captured
- *Stable*

Existing methods

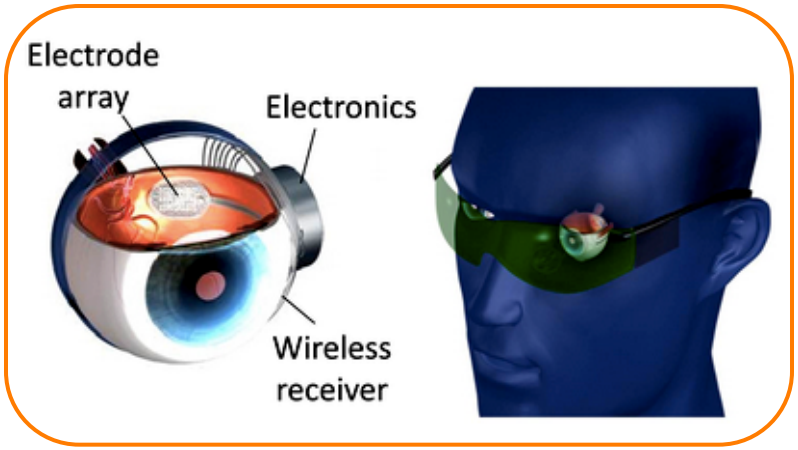
SUB-CELL techniques

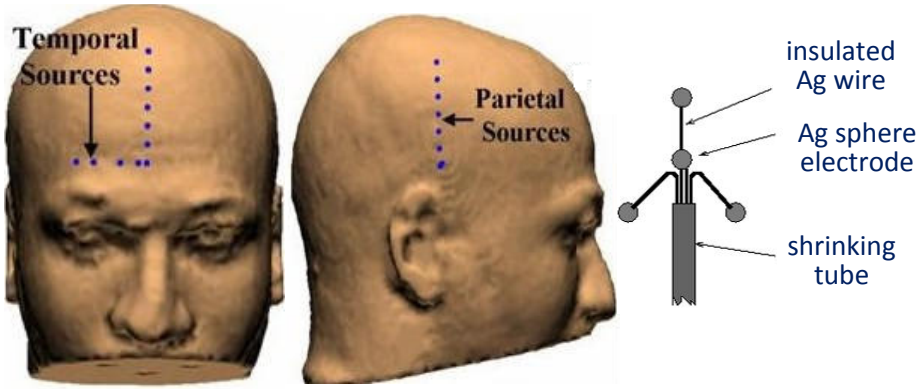
- Contour path FDTD (for thin wires)
- ADE-FDTD
- Sub-gridding techniques
- **DM-FDTD method**

GENERALIZATION

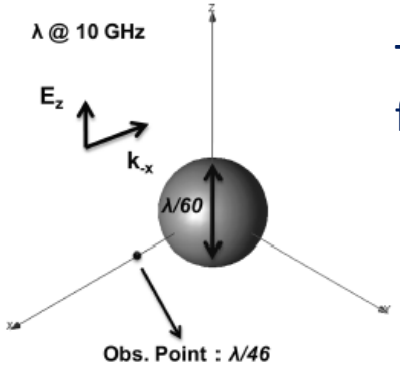
Full domain HYBRID methods

- Sub-gridding techniques
- ADI-FDTD
- FDTD + Marching On in Time (MOT) methods
- **MoM + FTDT** (iteration + Fourier transform)



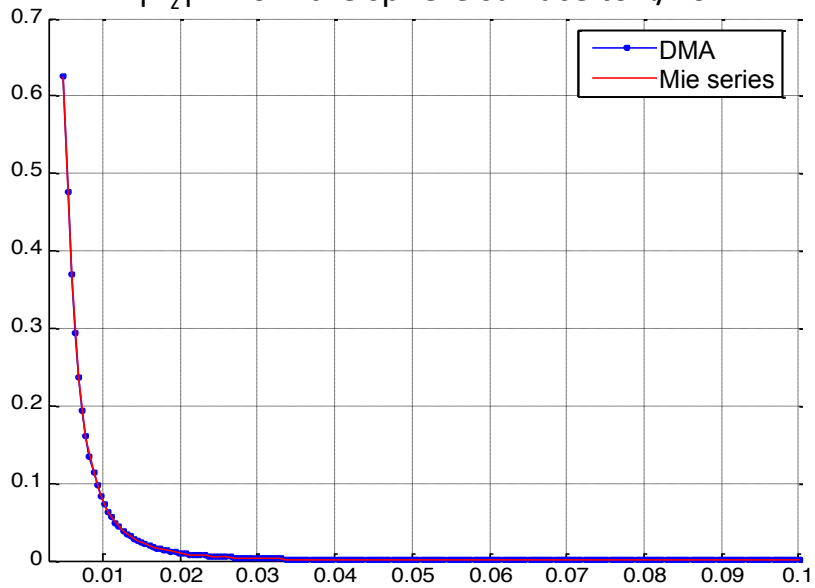


Dipole Moment method FD

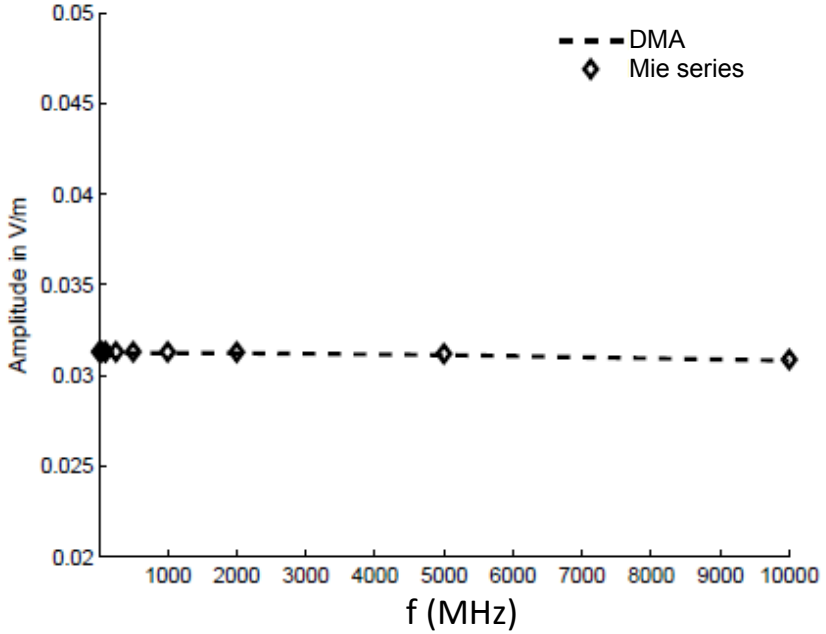


The fields scattered by an electrically small sphere, both in the near- and far-field can be equally derived with the DM method or the Mie series

|E_z| - from the sphere surface to λ/10



Magnitude of E_z



Quasi-static DM method (FD)

When the overall size of the problem geometry is small in comparison to the operating wavelength, i.e. $r \rightarrow 0\lambda$, then the $1/r^3$ term will dominate in the matrix terms computation:

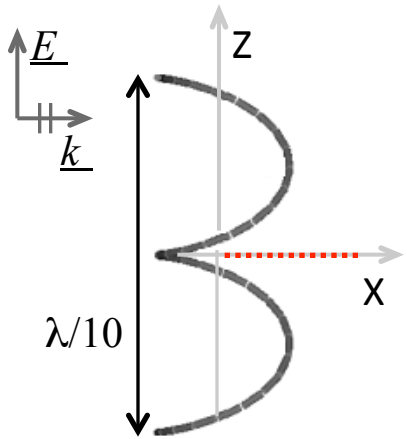
$$E_r = \frac{\eta I l}{2\pi}\left(\frac{1}{r^2}+\frac{1}{jkr^3}\right)e^{-jkr}\cos\theta \qquad E_\theta = \frac{j\eta I l}{4\pi}\left(\frac{k}{r}+\frac{1}{jr^2}+\frac{1}{kr^3}\right)e^{-jkr}\sin\theta$$

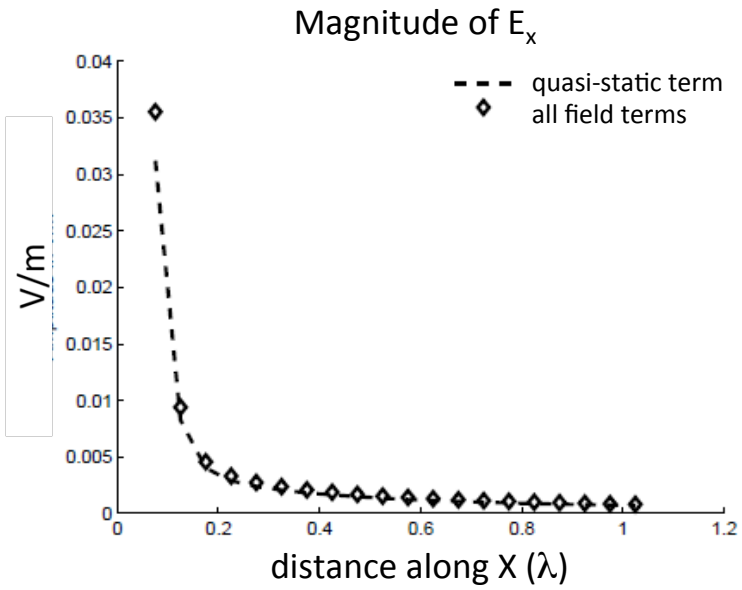
$r \rightarrow 0\lambda$

$$E_r = \frac{\eta I l}{2\pi}\left(\frac{1}{jkr^3}\right)\cos\theta \qquad E_\theta = \frac{j\eta I l}{4\pi}\left(\frac{1}{kr^3}\right)\sin\theta$$

Il

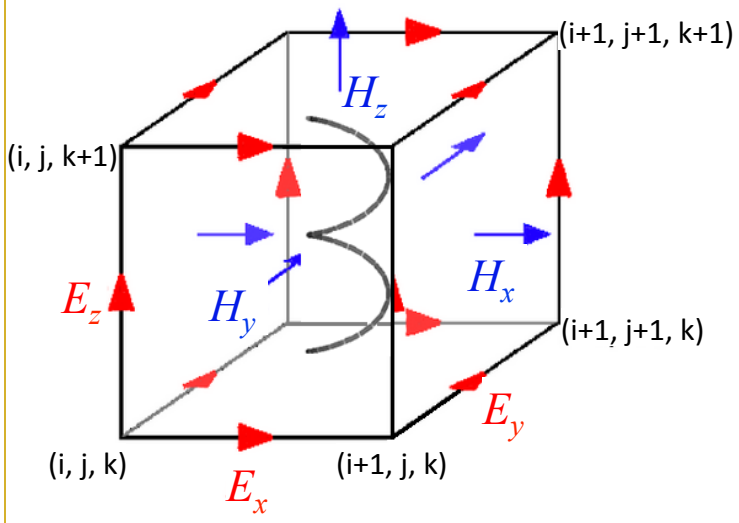
$$E_r = 2E_0\left(\frac{a}{r}\right)^3\cos\theta \qquad E_\theta = E_0\left(\frac{a}{r}\right)^3\sin\theta$$





Hybrid DM-FDTD sub-cell

The electric fields are real and time-independent we can use this to easily/efficiently hybridize DM with FDTD with minor changes to the update equations:



UPDATING SCHEME

- Assume we have found $\left(\underline{E}^{scat}\right)^{n}_{l,m,n}, \left(\underline{E}^{tot}\right)^{n}_{l,m,n}$ and $\left(\underline{H}^{tot}\right)^{n+1/2}_{l+1/2,m,n+1/2}$
- Increment time to $n+1$
- Update $\underline{E}$-field:

if $(i,j,k)\neq(l,m,n)$ → update $\underline{E}^{tot}$ conventional way

else → update equation at cell (l,m,n):

$$(\underline{E}^{inc})^{n+1}=(\underline{E}^{tot})^{n}-(\underline{E}^{s})^{n}+\Delta t/\varepsilon(\nabla\times\underline{H}^{tot})^{n+1/2}-\Delta t/\varepsilon(\nabla\times\underline{H}^{s})^{n+1/2}$$
$$=(\underline{E}^{tot})^{n}-(\underline{E}^{s})^{n}+\Delta t/\varepsilon(\nabla\times\underline{H}^{tot})^{n+1/2}-[(\underline{E}^{s})^{n+1}-(\underline{E}^{s})^{n}]$$
$$=(\underline{E}^{tot})^{n}-(\underline{E}^{s})^{n}+\Delta t/\varepsilon(\nabla\times\underline{H}^{tot})^{n+1/2}$$

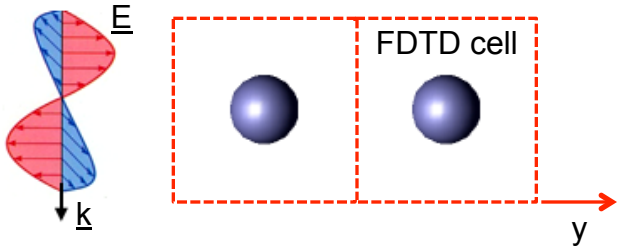
Update $(\underline{E}^{s})^{n+1}$ [DM field], and $(\underline{E}^{tot})^{n+1}=(\underline{E}^{scat})^{n+1}+(\underline{E}^{inc})^{n+1}$

- Increment time to $n+3/2$
- Update $\underline{H}$-field as usual (no modifications)

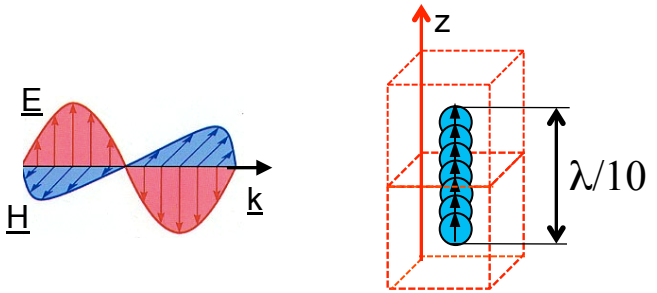
- This scheme only requires an updating of the total electric fields nearest the object contained within the FDTD cell
- It simply adds the DM scattered fields to the incident field at the FDTD nodal points, thus avoiding any contour path integration or the need to use auxiliary updating equations

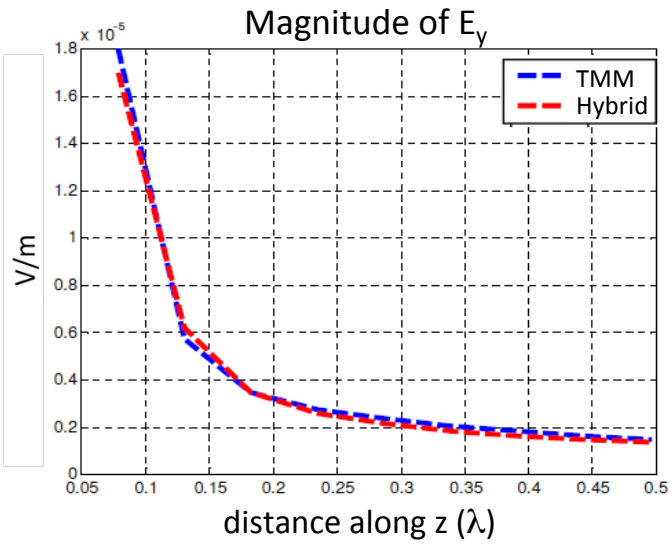
Hybrid DM-FDTD sub-cell

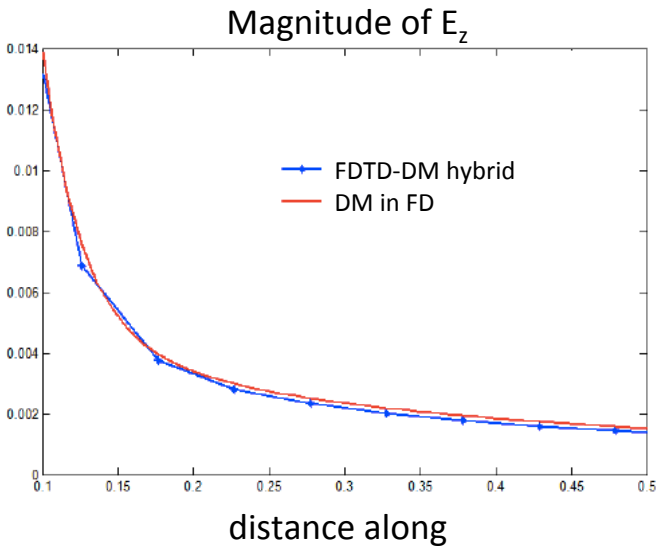
2 dielectric spheres



Small PEC wire traversing 2 cells







Limitation: The object must be electrically small (fit into 1-2 FDTD cells)

Dipole Moment method (FD)

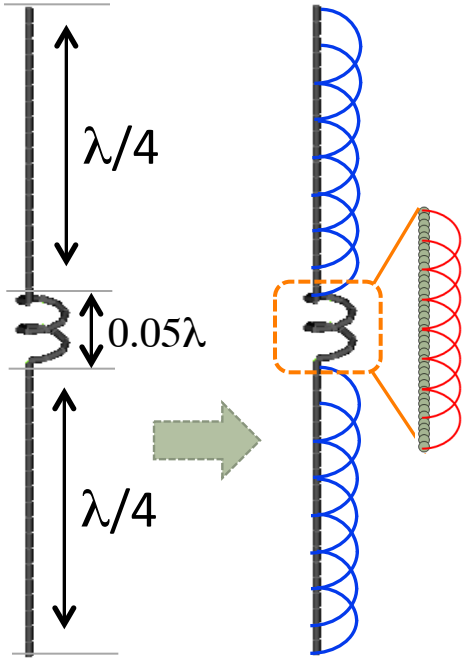
By using the DMs as basis functions to describe the current distribution, a matrix equation can be constructed by applying the appropriate boundary condition and we solve for the weights of the DMs

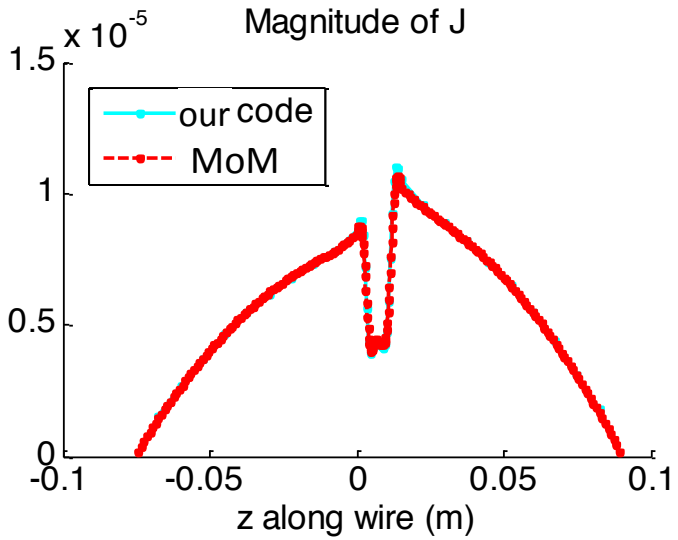
$$Il_x = E_0 \frac{4\pi j}{\eta k^2}(ka)^3 \qquad Il_x = E_0 \frac{4\pi j}{\eta k^2}(ka)^3\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)$$

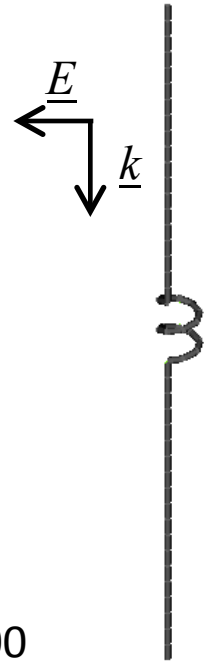
$$Kl_y = E_0 \frac{2\pi}{jk^2}(ka)^3 \qquad Kl_y = E_0 \frac{2\pi}{jk^2}(ka)^3\left(\frac{\mu_r - 1}{\mu_r + 2}\right)$$

$$E_r = \frac{\eta Il}{2\pi}\left(\frac{1}{r^2} + \frac{1}{jkr^3}\right)e^{-jkr}\cos\theta$$

$$E_\theta = \frac{j\eta Il}{4\pi}\left(\frac{k}{r} + \frac{1}{jr^2} + \frac{1}{kr^3}\right)e^{-jkr}\sin\theta$$

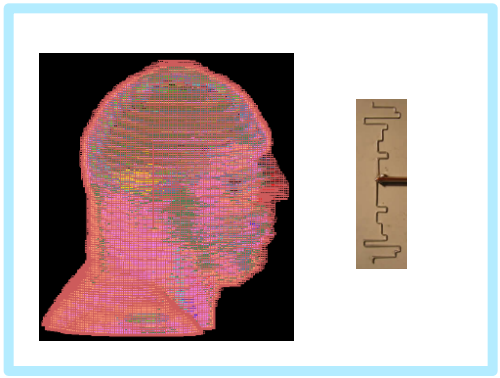






Comm. MoM: 211 segments Matrix CN: ~7000
My code: 15 unknowns Matrix CN: ~300

Hybrid MoM-FDTD method

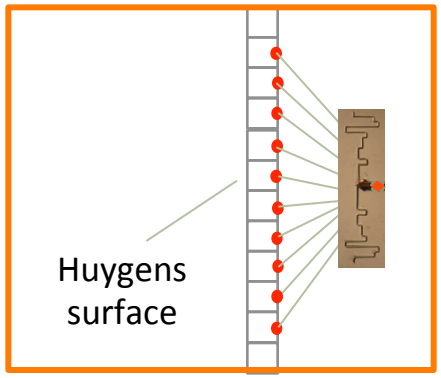


FINISH

Y

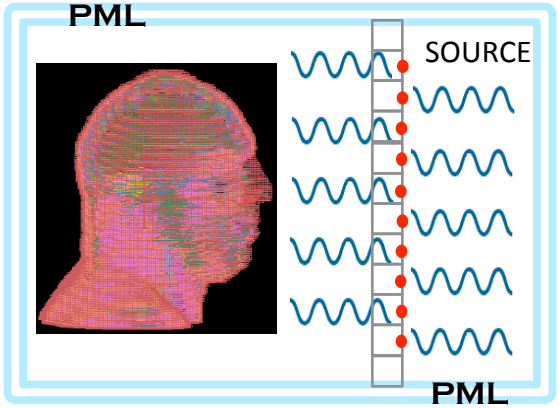
N

$J_n - J_{n-1} < t$



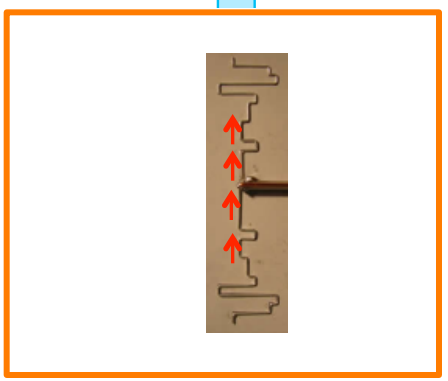
STEP 1

- ✓ MoM domain
- ✓ Large scatterer not present



STEP 2

- ✓ FDTD domain
- ✓ Source antenna not present

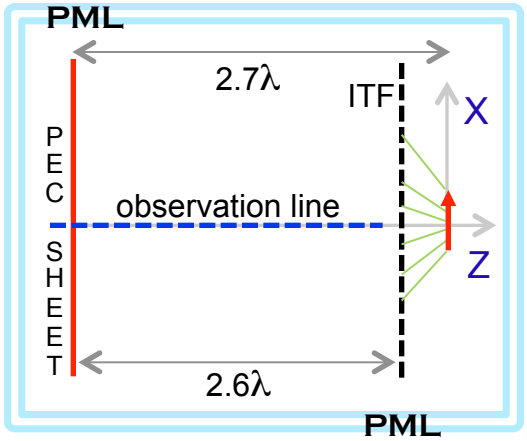


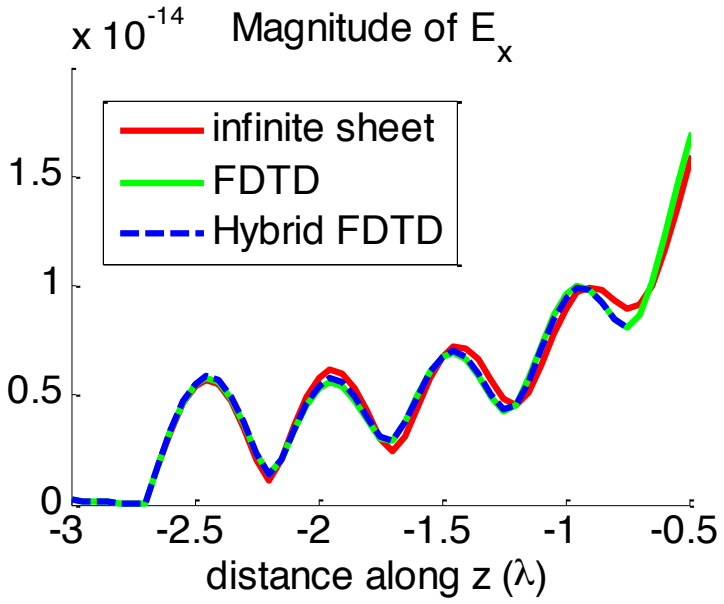
STEP 3

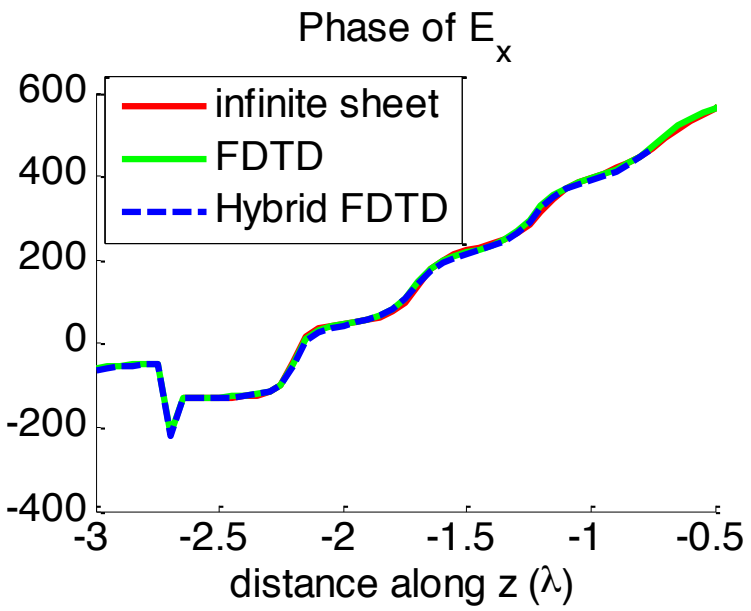
- ✓ MoM domain
- ✓ Large scatterer not present

Numerical Examples:
Small dipole w/ PEC sheet

- Cell size: λ/20 @ 10GHz; ITF size is 2.5λ
- Source dipole located at the origin, oriented along X
- Observation line along z from -3λ to -0.5λ @X=Y=0
- PEC sheet location: 2.7λ away from the source dipole

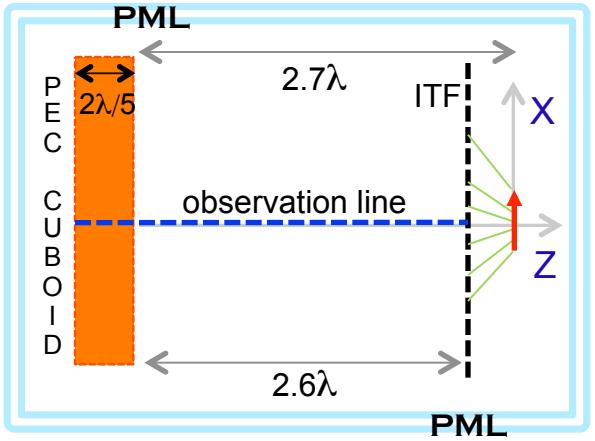


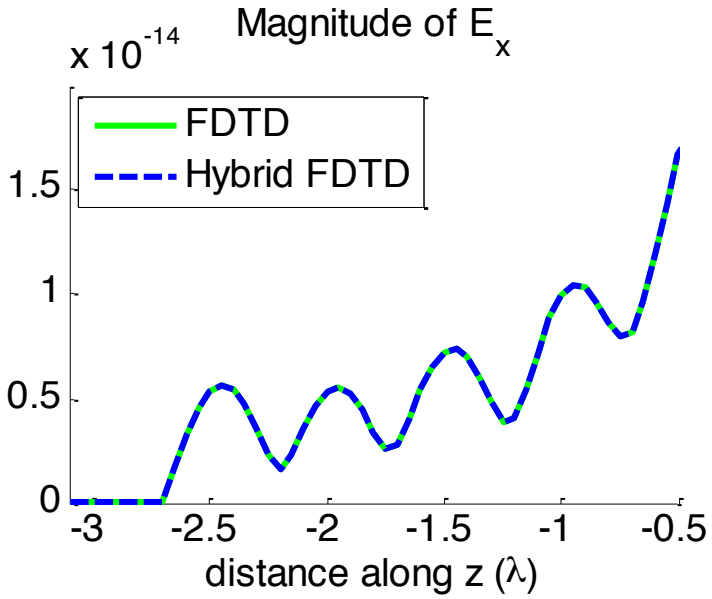


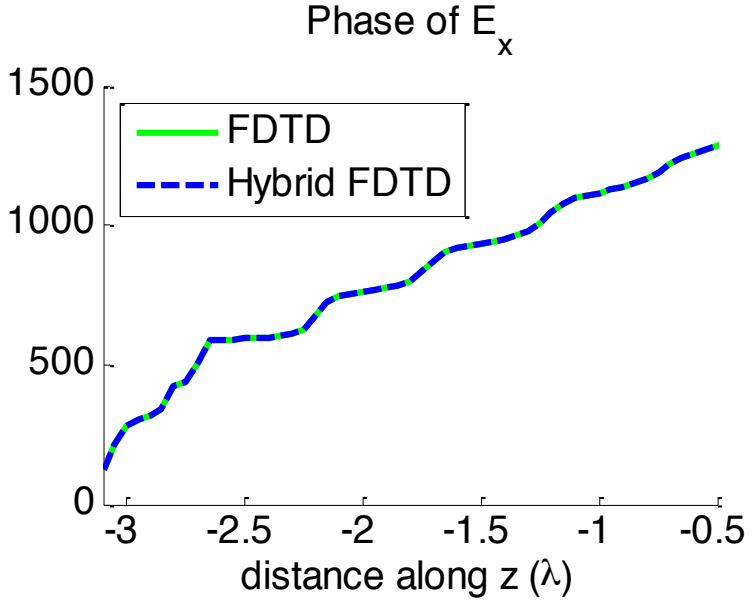


Numerical Examples:
Small dipole w/ PEC cuboid

- Cell size: λ/20 @ 10GHz;
- Source dipole located at the origin, oriented along X
- Observation line along z from -3λ to -0.5λ @X=Y=0
- PEC sheet location: 2.7λ away from the source dipole

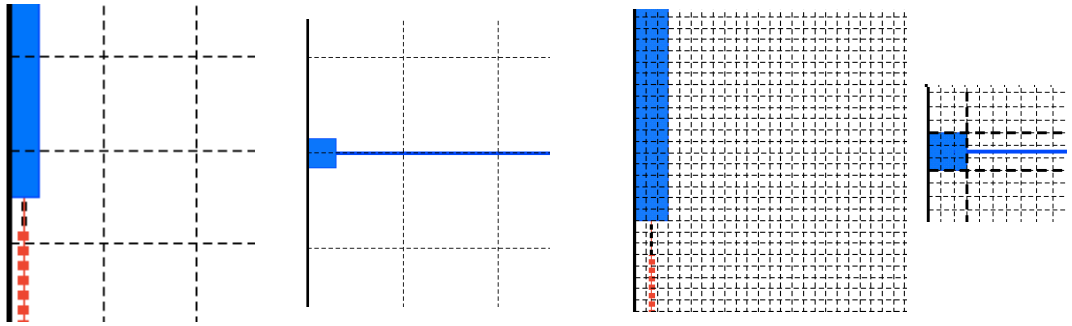


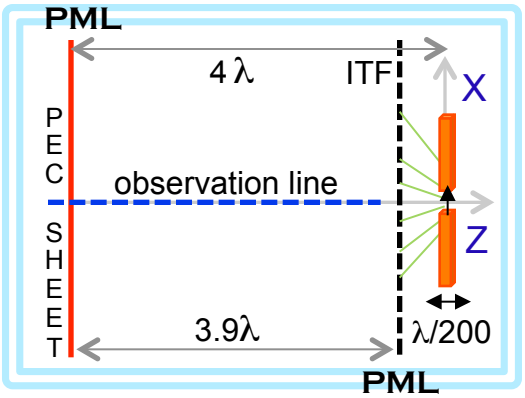


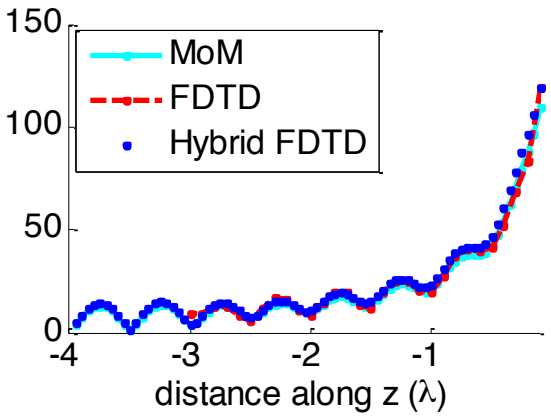


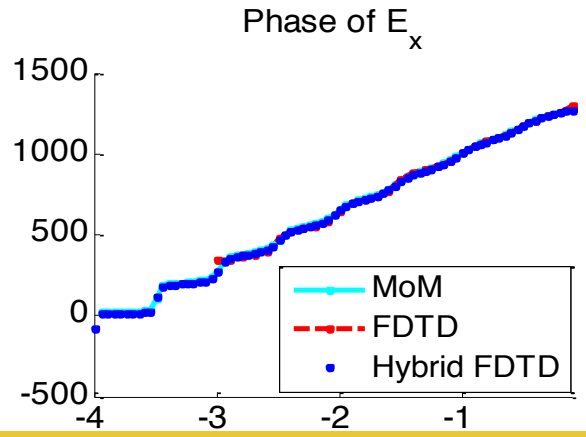
Numerical Examples:
Resonant dipole w/ PEC sheet

- Cell size: λ/20 @ 10GHz
- Interface size: 2.5λ×2.5λ
- Observation line along z from -4λ to 0 @X=Y=0
- PEC sheet location: 4λ away from the source dipole





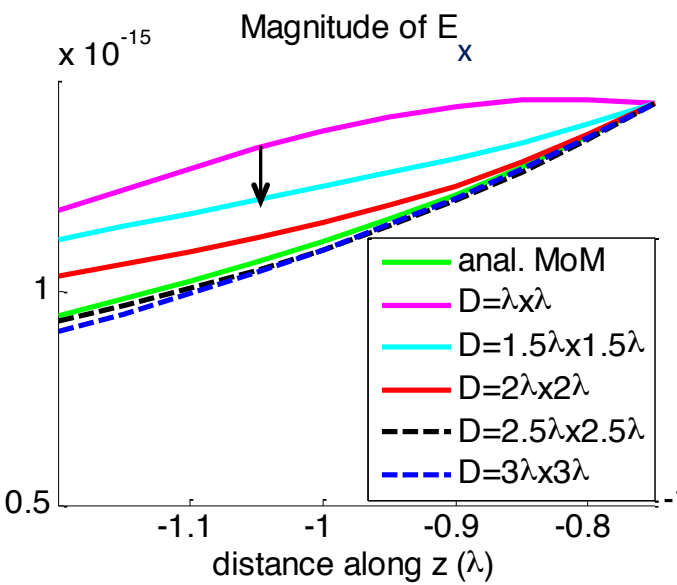


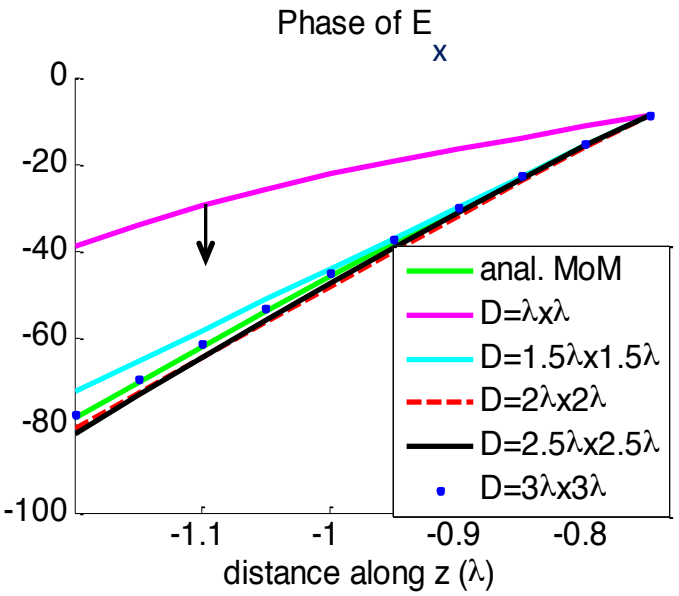


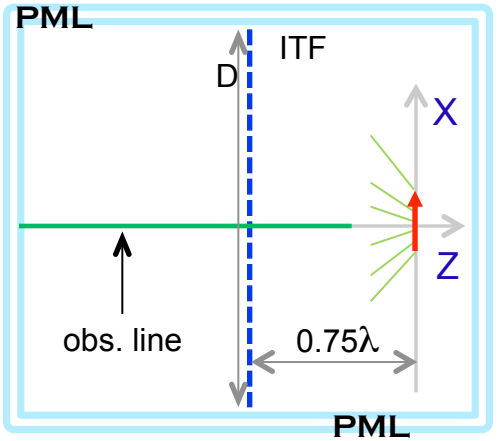
Non-uniform FDTD mesh RATIO: fine/coarse	
Time	5.42
Memory	8.57
Mcells	15

Numerical considerations:
Size of interface in terms of wavelengths

- Cell size: λ/20 @ 10GHz;
- Source antenna located at the origin
- The size D of the interface is varied: λ-1.5λ-2λ-2.5λ-3λ
- The fields are compared over a line at 10 GHz

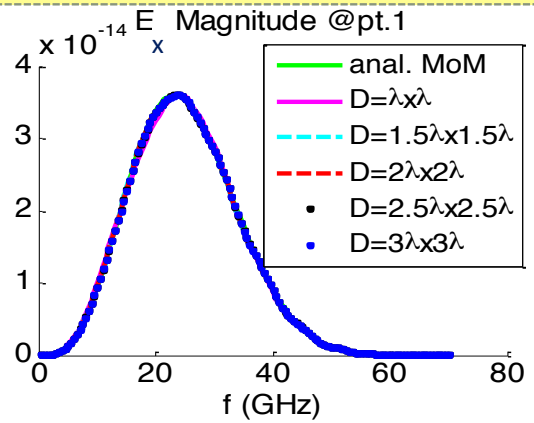


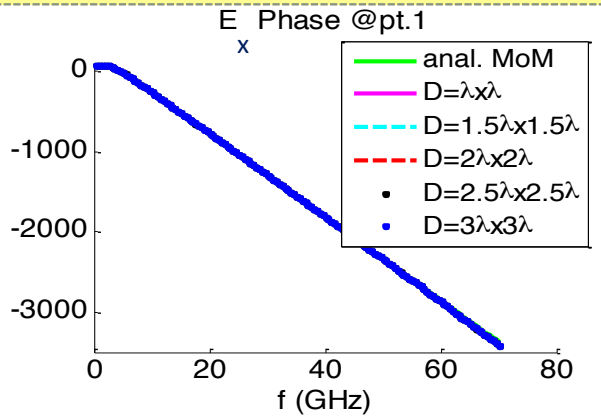


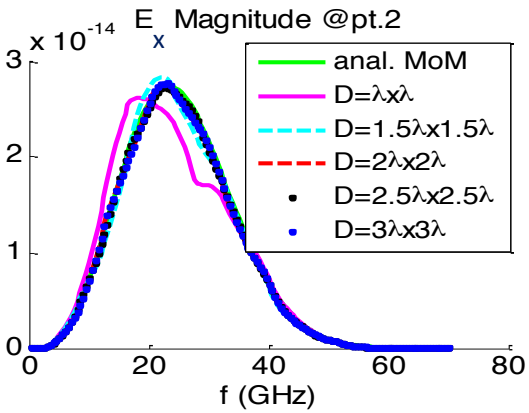


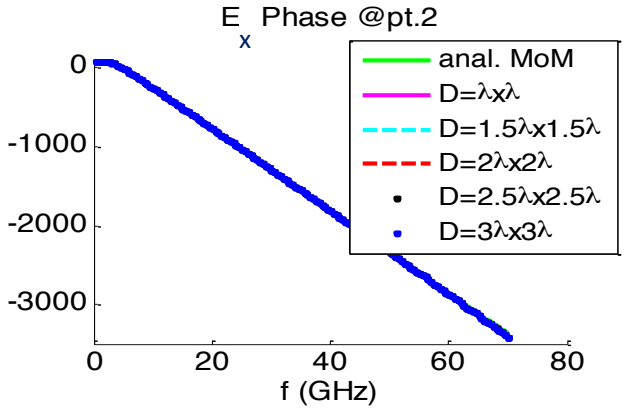
Numerical considerations:
Size of interface in terms of wavelengths

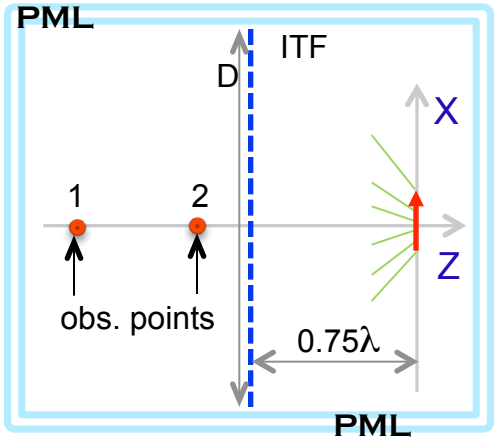
- Cell size: λ/20 @ 10GHz;
- Source antenna located at the origin
- The size D of the interface is varied: λ-1.5λ-2λ-2.5λ-3λ
- The fields are compared at points 1 and 2: 6 cells and 1 cell away from the interface as a function of *f*





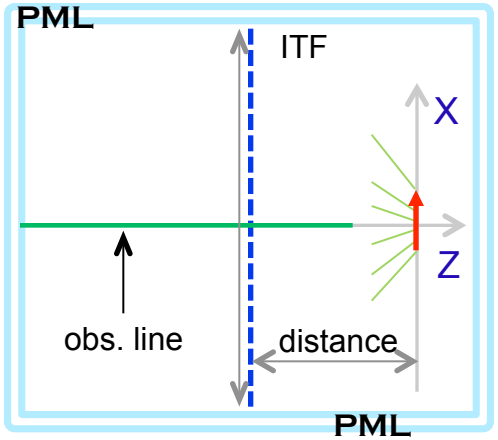




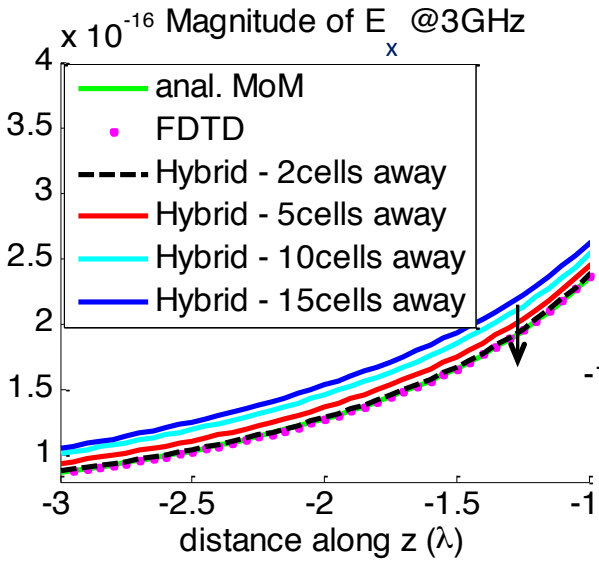


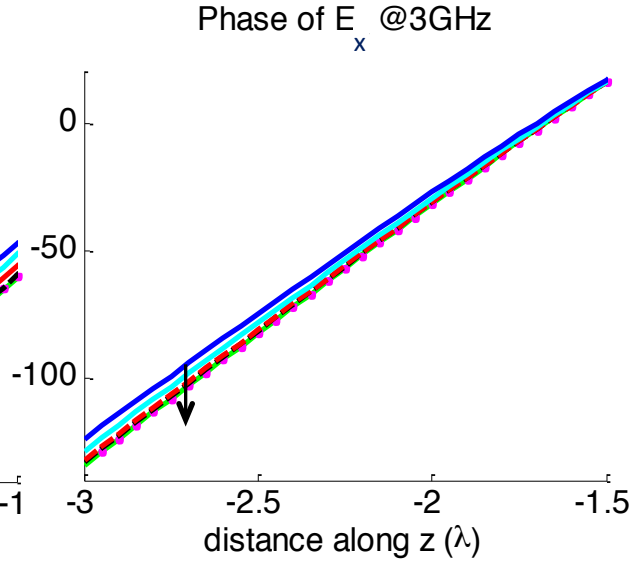
Numerical considerations:
Distance of interface from source

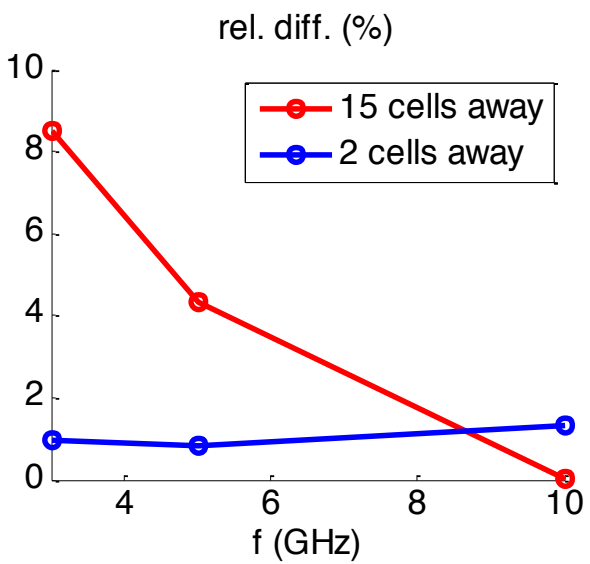
- Cell size: λ/20 @ 10GHz
- Interface size λ×λ
- Source antenna located at the origin
- The interface is placed 2, 5, 10, 15 cells from the source
- The fields are compared over a line at 5 GHz



$$rel.\,diff.(\%) = \left(\frac{|E_y\,FDTD| - |E_y\,Hybrid|}{|E_y\,FDTD|} \right)$$

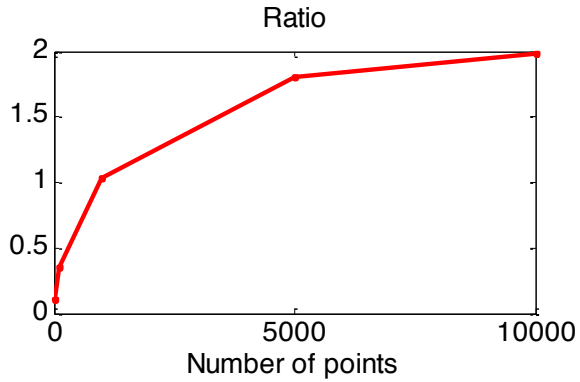




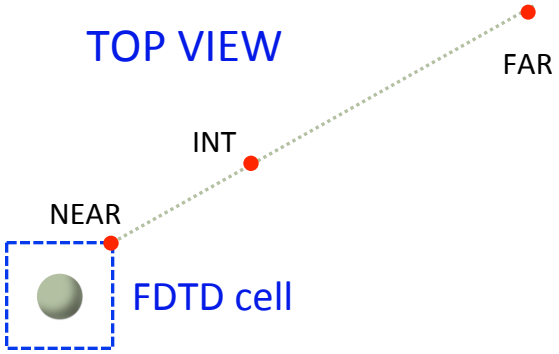


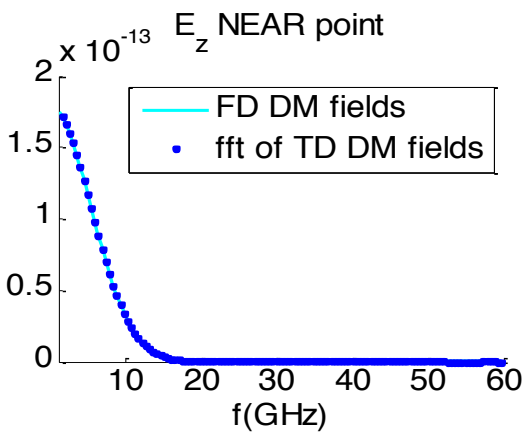
Numerical considerations: A method to eliminate the IFT operation

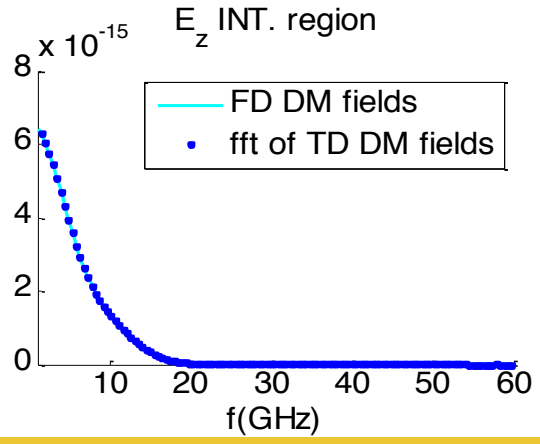
- Single DM source, 1GHz
- Gaussian source, the scattered fields in FD are computed 2 ways
  1. Get $E^{inc}(f)$ → compute $E^{scat}(f)$ with DM in FD
  2. Compute $E^{scat}(t)$ with DM in TD (DMTD) → take FT: $E^{scat}(f)$

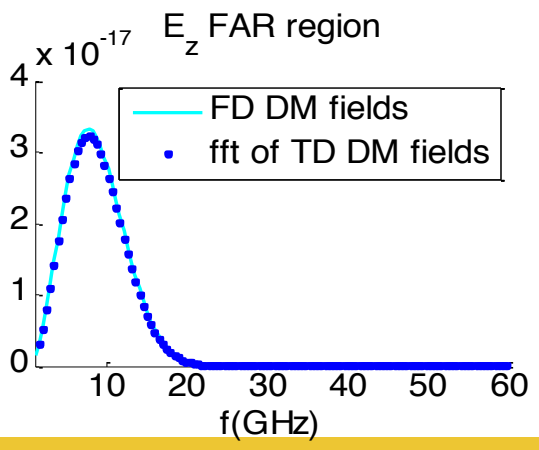


$$RATIO = 10\log\left(\frac{time\ IFT}{time\ DM\ TD}\right)$$

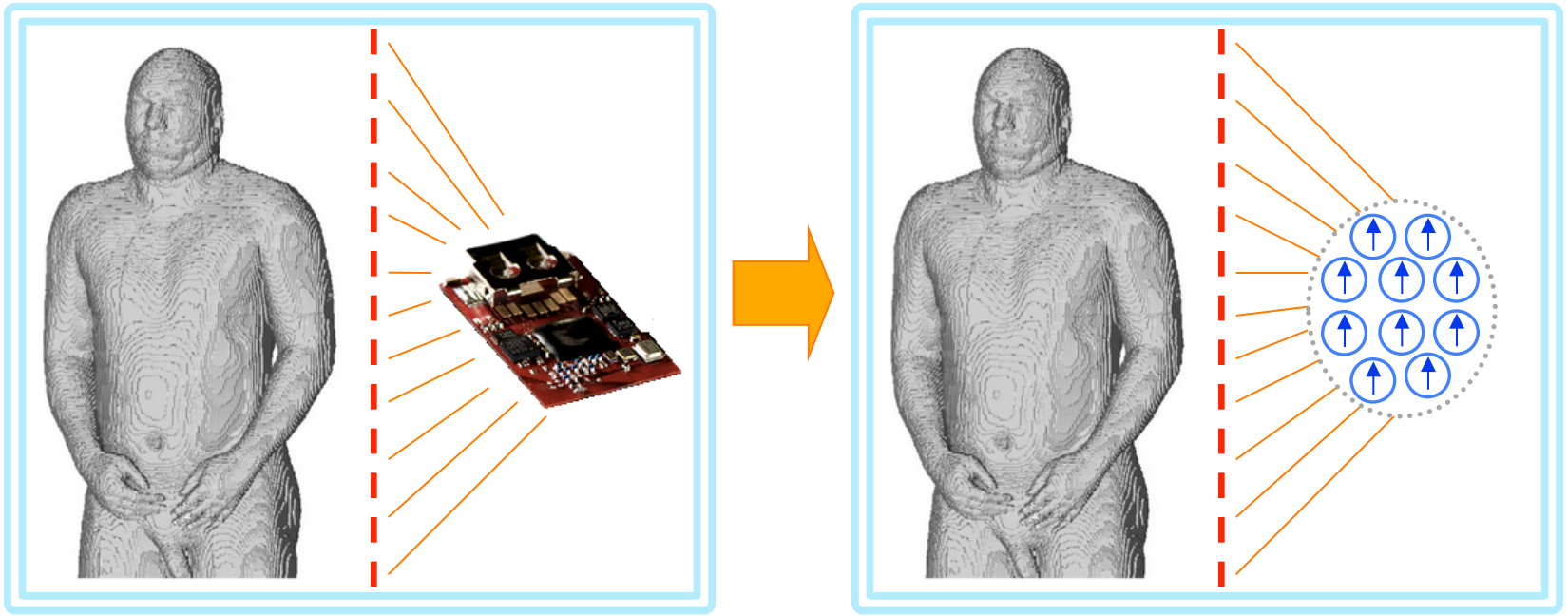






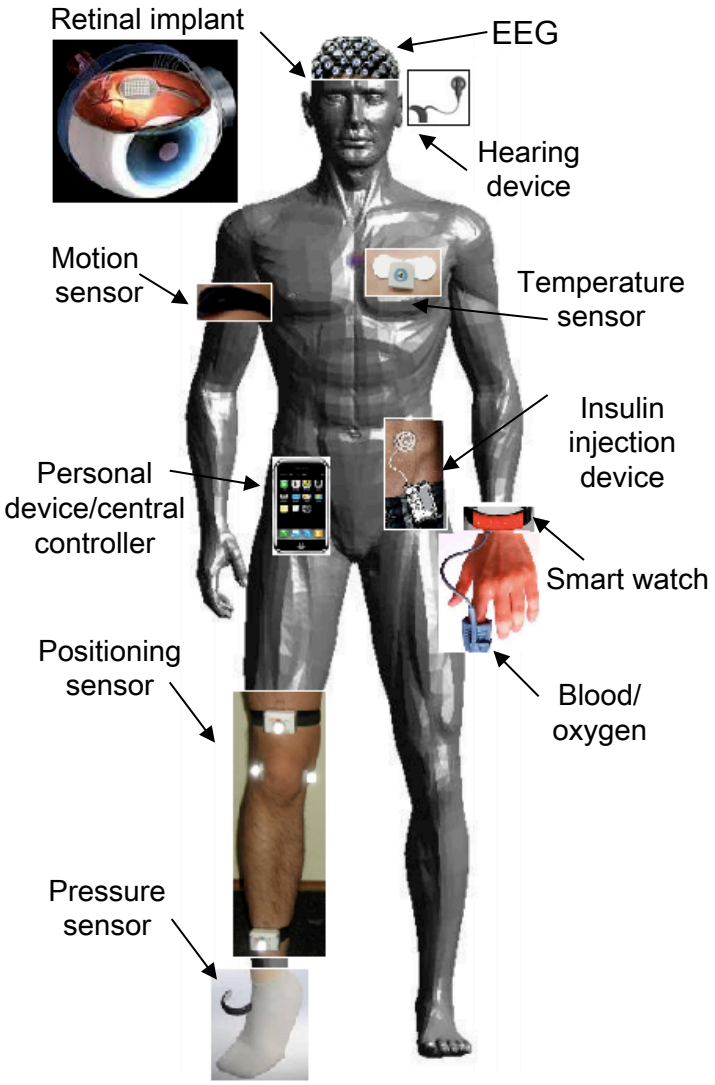


Future developments



- Fit the field over the interface with a collection of Dipole Moments
- Use them to get the fields over the interface in the time domain directly from the incident field on the antenna at every iteration step

Conclusions and Future work



- Review and limitations of sub-cell DM-FDTD method
- Analytical computation of the fields radiated from the antenna with MoM
- Generalized MoM-FDTD algorithm to model arbitrary wire element with curvature, not aligned with the FDTD grid in the presence of an electrically large object
- Method for bypassing the time-consuming IFT operation at the interface

Further developments

- Generalization of the method for bypassing the IFT operation at the interface to arbitrary antennas